Thinking & Reasoning

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/ptar20

The new paradigm and massive modalization

David E. Over

To cite this article: David E. Over (2021): The new paradigm and massive modalization, Thinking & Reasoning, DOI: 10.1080/13546783.2021.2017346

To link to this article: https://doi.org/10.1080/13546783.2021.2017346

Published online: 30 Dec 2021.

Submit your article to this journal

Article views: 41

View related articles

View Crossmark data

Full Terms & Conditions of access and use can be found at
https://www.tandfonline.com/action/journalInformation?journalCode=ptar20

THINKING & REASONING
https://doi.org/10.1080/13546783.2021.2017346

Routledge
Taylor & Francis Group

Check for updates

The new paradigm and massive modalization

David E. Over

Psychology Department, University of Durham, Durham, UK

ABSTRACT
Knauff and Gazzo Castañeda argue as much in support of revised mental model theory (RMMT) as they argue against talk of a new paradigm caused by the probabilistic approach in the psychology of reasoning. They claim that RMMT is not essentially different from classical mental model theory (CMMT) and not essentially different from the probabilistic approach. There are many serious questions to ask about RMMT. But RMMT is a massive modalization of aspects of the extensional CMMT, and it follows the probabilistic approach in having an intensional focus that justifies talk of a new paradigm.

ARTICLE HISTORY Received December 1, 2021; Accepted December 6, 2021

KEYWORDS New paradigm; probabilistic approach; mental model theory; extensional; intensional; modalized

Knauff and Gazzo Castañeda have written a stimulating critique of talk of a new paradigm in the study of reasoning. They have also presented *revised mental model theory* (RMMT), first proposed by Johnson-Laird et al. (2015), in a positive light. In fact, they try to promote RMMT as much as they try to deflate new paradigm talk. Their article concludes that the new paradigm is not radically different from RMMT, and that RMMT is not essentially different from *classical mental model theory* (CMMT), founded by Johnson-Laird and Byrne (1991). To avoid confusion, it will be best from now on to refer to what has been called the new paradigm as the *probabilistic approach* in the psychology of reasoning. It has also been called the Bayesian approach (Oaksford & Chater, 2007, 2020; Over, 2009, 2020; Over & Cruz, 2018). I will argue that serious questions about RMMT must be answered before its relation to the probabilistic approach can be clarified, but RMMT is a radical change from CMMT and itself justifies talk of a new paradigm.

CONTACT David E. Over david.over@durham.ac.uk Psychology Department, University of Durham, South Road, Durham DH1 3SL, UK.

© 2022 Informa UK Limited, trading as Taylor & Francis Group

The conditional modalized

At the most fundamental logical level, the probabilistic approach differs essentially from CMMT in proposing *intensional* theories, rather than an *extensional* one, and a different logic for the natural language conditional, *if p then q*. As Knauff and Gazzo Castañeda recall, CMMT was committed to the supposed logical validity of inferring such a conditional from *not-p* and from *q*. These inferences are the *paradoxes* of the *material conditional* (implication). They result from claiming that the natural language *if p then q* is logically equivalent to the material conditional, *not-p or q*, in classical extensional logic. It is logically valid in this logic to infer *not-p or q* from *not-p* and from *q*, using the fundamental inference of *or-introduction*. According to CMMT, *if p then q* has the same full mental models as the material conditional, making the paradoxes logically valid in CMMT (Johnson-Laird & Byrne, 1991, p. 74). There are many logical and philosophical arguments against the supposed logical "validity" of the paradoxes, and the weight of evidence in the psychology of reasoning against this claimed "validity," as following from people's understanding of *if p then q*, has grown and grown (Evans & Over, 2004; Over, 2020). CMMT had to be abandoned as a scientific research programme (Lakatos, 1978), and as in a true paradigm shift, all its followers converted in a flash to RMMT, radically changing their beliefs, perhaps before breakfast, like the White Queen in *Through the Looking-Glass*.

In the probabilistic approach, it is invalid to infer *if p then q* from *not-p* and from *q*. Its logic is based on the intensional notion of probability (Adams, 1998). The fact that this new approach changes the logic of the conditional is a good reason to start talking about a new paradigm. There is much more that is new in the probabilistic approach, especially in its focus on the intensional notions of belief, degrees of belief and so subjective probabilities, and belief updating (Oaksford & Chater, 2020; Over, 2020). But if a change in the logic of such a fundamental logical concept as that of the conditional is not a paradigm change, what could be? The only extreme revision that could make a better case for talk of a new paradigm would be if the valid inferences were changed for more than one fundamental logical concept, and that is exactly what we find in RMMT. It proposes to alter the essential logic of both *or* and *if*, of both disjunction and the conditional, and as we shall see, the RMMT change in the logic of *or* implies a change in the classical meaning of *and*, and possibly of *not*, as well.

As Knauff and Gazzo Castañeda explain, CMMT made *if p then q* equivalent to a disjunction, essentially to the extensional disjunction *(p & q) or (not-p & q) or (not-p & not-q)*, when fully represented. RMMT changes the default meaning of *if p then q*, and *not-p or q*, to a *modalized* and so intensional conjunction, *possible (p & q) & possible (not-p & q) & possible (not-p & not-q)*. But an extremely important fact that Knauff and Gazzo Castañeda do not report is that *impossible (p & not-q)* must be added, at least implicitly, to this conjunctive list (Johnson-Laird et al., 2015; Khemlani et al., 2018). Now not only does *if* become an *and* in RMMT, but *or* becomes an *and* in RMMT as well. As far as I know, *not* has not become an *and* in RMMT, which is a mercy. But whatever is true about the RMMT meaning of *not*, RMMT radically changes the extensional conditionals and disjunctions of CMMT into modal conjunctions. This massively modalized proposal, employing what is supposed to be epistemic possibility, is more than enough to justify talk of RMMT as a new paradigm compared to CMMT. Questions about RMMT make its relation to the probabilistic approach unclear, and I will come to these after going into detail about the RMMT treatment of disjunction (see Baratgin et al., 2015, for the first criticisms of RMMT, and Wang et al., in press, and Wang & Zheng, 2021, for disconfirmation of RMMT on people's representations of conditionals and disjunctions).

Disjunction modalized

Consider two fair coins, one gold and one silver, that are tossed independently and randomly and fall on a tabletop, and this disjunction about these coins:

(1) The gold coin is showing a head (*p*) or the silver coin is showing a head (*q*).

In CMMT, the full representation of (1) was equivalent to the following extensional disjunction:

(2) *(p & q) or (p & not-q) or (not-p & q)*

In RMMT, the full representation of (1) is equivalent to the following default intensional and modalized conjunction:

(3) *possibly (p & q) & possible (p & not-q) & possible (not-p & q) & impossible (not-p & not-q)*

Khemlani et al. (2018) present (3) as a "vertical," rather than a "horizontal," conjunction, but no matter how (3) is written, it is the full representation of the meaning of (1) in RMMT.

The logical difference between (2) and (3) as semantic contents for (1) could not possibly be exaggerated. It is the difference between an extensional disjunction, (2), in CMMT and a modalized conjunction, (3), in RMMT. Or-introduction fails to be valid in RMMT (Johnson-Laird et al., 2015) because (1) and (3) are taken to be equivalent (see Cruz et al., 2017, on serious problems with, and disconfirmation of, the RMMT claim that or-introduction is "invalid"). We will ask below what exactly "valid" means in RMMT, but not even *possible (p & q)*, the first conjunct of (3), validly follows from *p* in standard modal logic, as *q* can be impossible when *p* is true. On the other hand, (1) does validly follow from *p* when (1) is equivalent to (2). Either *(p & q)* or *(p & not-q)* is true when *p* is true in classical logic and CMMT, and (1) and (2) validly follow from *p* in the probabilistic approach, for the probability of *p* cannot be coherently higher than the probability of (1) and of (2). Probability theory absolutely depends on the classical meanings of *not*, *and*, and *or* and so on the classically valid introduction and elimination rules for these logical operators.

The probabilistic approach and CMMT do not diverge from each other on the meanings of *not*, *or*, and *and*. But we can now illustrate the problems of coherently relating RMMT to the probabilistic approach, despite what Knauff and Gazzo Castañeda argue. Given our assumptions about the coins and how they got onto the tabletop, the probability of (1), *P(p or q)*, is *P(p & q)* + *P(p & not-q)* + *P(not-p & q)* = .75, and we can also infer in probability theory that *P(not-p & not-q)* = .25. But if (*not-p & not-q)* has a probability of .25, then (*not-p & not-q)* is not impossible, and if *(not-p & not-q)* is clearly not impossible, then (3) is definitely false. That appears to imply, totally counterintuitively, and certainly in conflict with the probabilistic approach, that (1) has 0 as its probability in RMMT for our example.

I say that the probability of (1), when it is supposed to be equivalent to (3), "appears" to be 0 under certain conditions, because it is unclear what account can be given of the probability of (3). It is unclear what account can be given of a simpler combination of probability and possibility, *P(possible p)*. We say that it is improbable we will win a lottery *l*, and this clearly means that *P(l)* is low. But *P(possible l)*, the probability that it is possible that we will win the lottery, would seem to be 1, if it makes sense at all. Referring to Khemlani and Johnson-Laird (in press), Knauff and Gazzo Castañeda say that RMMT and probability theory have been "… integrated into a unified theory …" But I find no explanation in that article of what *P(possible p)* is supposed to mean, as a first step to explaining what it can mean to speak of the probability of (1), when (1) is supposed to be logically equivalent to (3) and not (2).

There is also the problem of the logical relation between *or* and *and*, and perhaps *not*, that I referred to above. Suppose that both *p* and *(not-p & not-q)* hold at the same time. It is logically valid by &-elimination to infer *not-p* from *(not-p & not-q)*, giving us a contradiction between *p* and *not-p*. Therefore, when *p* holds, *not-(not-p & not-q)* follows by the logical validity of the *reductio ad absurdum* inference. But *not-(not-p & not-q)* is logically equivalent to *p or q* in classical logic and in CMMT, and this equivalence means that we can validly infer *p or q* from *p*, against the claim of RMMT that this inference, or-introduction, is "invalid." There are now two possibilities. First, RMMT is inconsistent. I will say more about this possibility below.

Second, at least one of these inferences is invalid in RMMT: &-elimination, *reductio ad absurdum*, or the equivalence. If one of these inferences is invalid in RMMT, RMMT has not only changed the fundamental logical meaning of *or*, but also of *and* (see Khemlani et al., 2018, on conjunctions), or *not*, or both. The RMMT "invalidity" of or-introduction makes it impossible to integrate RMMT and probability theory.

All the inferences I have mentioned in the previous paragraph are logically valid in the probabilistic approach, and *P(p or q)* = *P(not-(not-p & not-q))* in this approach. If this equivalence does not hold in RMMT, how is the "probability" of *(not-(not-p & not-q))* calculated in RMMT and what is it? Whatever it is, it can only be a "probability," and not a probability, as this RMMT "probability" cannot satisfy the axioms of probability theory.

Validity in RMMT

To be clear and precise about what "validity" means, and which inferences are logically valid, or invalid, in RMMT, it would help immensely if there were a logical model theory for RMMT. But though the proponents of RMMT always refer to "the" model theory (Johnson-Laird et al., 2015; Khemlani et al., 2018), they do not have a model theory in the sense in which logicians use the term. In this logical sense, CMMT does have a model theory. Suppose that we want to show that it is invalid in classical logic, and CMMT, to infer *p* from *p or q*. The models of *p or q*, in the logical sense, can be represented by the rows of the truth table in which *p or q* is true, and these are the full CMMT models for *p or q*. Consider the row in which *p* is false and *q* is true. In this row, *p or q* is true but *p* is false. There is thus a logical and CMMT model in which *p or q* is true and *p* is false, and that proves that it is invalid to infer *p* from *p or q* in classical logic and in CMMT, and in the probabilistic approach. On this basis, it is trivial to construct a probabilistic model in which *P(p or q)* = 1 and *P(p)* = 0.

But how does one prove *p* does not validly follow from *p or q* in RMMT? Knauff and Gazzo Castañeda claim that RMMT retains the "core" idea of CMMT that an inference is invalid if it has a counterexample. To produce this counterexample, we need a RMMT model in which *p or q* is true and *p* is false. But so far, supporters of RMMT have merely written (3) down vertically, as if this long modal conjunction somehow sufficed as a model in which *p or q* is true (Khemlani et al., 2018). They have not even given us a model in which *possible p* is true. They cannot make use here of the logical model theory of standard modal logic, by taking subsets of rows of truth tables as epistemic "possible worlds" in the technical sense. For RMMT supporters totally reject that logical model theory as relevant to their system of reasoning (see also Hinterecker et al., 2016). But they propose no alternative recursive definition of truth in a model, to define validity for their modalized system.

Logicians have severely criticised RMMT for its claims about logic and "model theory," and modal logic in particular (Bringsjord & Govindarajulu, 2020). A major problem for RMMT is that it does not have a soundness, or consistency, proof for its system of reasoning. There are proofs of soundness, and so consistency, and of completeness and decidability, for classical sentential logic, and for the sentential logic underlying the probabilistic approach (Adams, 1998). RMMT introduces new inferences for its massively modalized default representations, and it runs a risk in doing that (Oaksford et al., 2019). How do we know that the supposedly "valid" inferences of RMMT form a consistent system and have no counterexamples? There is an unpublished supposed consistency "proof" on the mental models website (Johnson-Laird, 2020), but it does not prove consistency in any way. It does not end in a logical model of the system, and it does not prove that RMMT is sound, with no counterexamples to one of its supposedly "valid" new inferences.

Conclusions

Knauff and Gazzo Castañeda cannot justifiably argue that there is smooth continuity between CMMT and RMMT. RMMT is a massively modalized new paradigm compared to the extensional CMMT. Knauff and Gazzo Castañeda are equally unjustified in claiming that RMMT can be "integrated" and "unified" with probability theory, which underlies the probabilistic approach. RMMT follows the intensional lead of the probabilistic approach, but it is unclear how the two approaches can have a closer relation than that.[1]

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

David E. Over http://orcid.org/0000-0002-0353-9381

References

Adams, E. (1998). *A primer of probability logic*. CLSI Publications.

[1]Thanks to Nicole Cruz, Jonathan Evans, and Mike Oaksford.

Baratgin, J., Douven, I., Evans, J., St, B. T., Oaksford, M., Over, D. E., & Politzer, G. (2015). The new paradigm and mental models. *Trends in Cognitive Sciences*, *19*(10), 547–548. https://doi.org/10.1016/j.tics.2015.06.013

Bringsjord, S., & Govindarajulu, N. S. (2020). Rectifying the mischaracterization of logic by mental model theorists. *Cognitive Science*, *44*(12), e12898. https://doi.org/10.1111/cogs.12898

Cruz, N., Over, D., & Oaksford, M. (2017). The elusive oddness of or-introduction. In G. Gunzelmann, A. Howes, T. Tenbrink, & E. Davelaar (Eds.), *The 39th Annual meeting of the cognitive science society*. Cognitive Science Society.

Evans, J., & Over, D. E. (2004). *If*. Oxford University Press.

Hinterecker, T., Knauff, M., & Johnson-Laird, P. N. (2016). Modality, probability, and mental models. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *42*(10), 1606–1620. https://doi.org/10.1037/xlm0000255

Johnson-Laird, P. N. (2020). A proof of the consistency of the mental model theory of sentential reasoning. http://www.modeltheory.org/publications/

Johnson-Laird, P. N., & Byrne, R. M. J. (1991). *Deduction*. Hove & Hammersmith, Erlbaum.

Johnson-Laird, P. N., Khemlani, S., & Goodwin, G. P. (2015). Logic, probability, and human reasoning. *Trends in Cognitive Sciences*, *19*(4), 201–214. https://doi.org/10.1016/j.tics.2015.02.006

Khemlani, S. S., Byrne, R. M. J., & Johnson-Laird, P. N. (2018). Facts and possibilities: A model-based theory of sentential reasoning. *Cognitive Science*, *42*(6), 1887–1924. https://doi.org/10.1111/cogs.12634

Khemlani, S., & Johnson-Laird, P. N. (in press). Reasoning about properties: A computational theory. *Psychological Review*. https://doi.org/10.1037/rev0000240

Lakatos, I. (1978). *The methodology of scientific research programmes.* (Philosophical papers: Volume 1). Cambridge University Press.

Oaksford, M., & Chater, N. (2007). *Bayesian rationality: The probabilistic approach to human reasoning*. Oxford University Press.

Oaksford, M., & Chater, N. (2020). New paradigms in the psychology of reasoning. *Annual Review of Psychology*, *71*, 305–330.

Oaksford, M., Over, D. E., & Cruz, N. (2019). Paradigms, possibilities, and probabilities: Comment on Hinterecker et al. (2016). *Journal of Experimental Psychology: Learning Memory and Cognition*, *45*, 288–297.

Over, D. E. (2009). New paradigm psychology of reasoning. *Thinking & Reasoning*, *15*(4), 431–438. https://doi.org/10.1080/13546780903266188

Over, D. E. (2020). The development of the new paradigm in the psychology of reasoning. In S. Elqayam, I. Douven, J. St. B. T. Evans, & N. Cruz (Eds.), *Logic and uncertainty in the human mind* (pp. 243–263). Routledge.

Over, D. E., & Cruz, N. (2018). Probabilistic accounts of conditional reasoning. In Linden J. Ball & Valerie A. Thompson (Eds.), *International handbook of thinking and reasoning* (pp. 434–450). Psychology Press.

Wang, M., Over, D. E., & Liang, L. (in press). What is required for the truth of a general conditional? *Quarterly Journal of Experimental Psychology*, https://osf.io/ume7q/

Wang, M., & Zheng, L. (2021). Does the inclusive disjunction really mean the conjunction of possibilities? *Cognition*, *208*, 104551.