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Short Communication

Does the inclusive disjunction really mean the conjunction of possibilities?

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ABSTRACT

There is an ongoing dispute in the psychology of reasoning about how people interpret disjunctions, *p or q*. In the original mental models theory (MMT1) people interpret *p or q* as the disjunction of three possibilities (possibly *p¬q*, or possibly *¬pq*, or possibly *pq,* where "¬" = not). *p or q* is true if one disjunct is actually true. In a recent revision of mental models theory (MMT2), people interpret *p or q* as a conjunction of the three possibilities, and they treat it as true only if each is possible and *¬p¬q* is impossible. Two experiments investigated possibility and truth judgments about disjunctions given sets consisting of one or more of the four cases (*p¬q*, *¬pq*, *pq*, and *¬p¬q*). The results showed that in both possibility and truth judgments, participants' interpretations of disjunctions were only consistent with MMT1. Inclusive disjunctions imply the disjunction of the three possibilities, and they are true when one of the three cases (*p¬q*, *¬pq*, and *pq*) is actual. These findings support MMT1, but not MMT2. In conclusion, the revised mental models theory may be unnecessary for disjunctions.

1. Introduction

A major current debate in the psychology of reasoning concerns how people understand disjunctions, *p or q*, for example, *Anna is an actor or a writer* (Baratgin et al., 2015; Hinterecker, Knauff, & Johnson-Laird, 2016; Hinterecker, Knauff, & Johnson-Laird, 2019; Johnson-Laird, Khemlani, & Goodwin, 2015; Oaksford, Over, & Cruz, 2019)? The proper interpretation of the disjunction divides older versions of mental models theory (MMT1) (Jeffrey, 1981; Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002), from a recently revised version of that theory (MMT2) (Hinterecker et al., 2016; Johnson-Laird et al., 2015; Khemlani, Byrne, & Johnson-Laird, 2018). It has been argued that MMT2 is theoretically problematic (Oaksford et al., 2019). In this paper, we present two experiments on the interpretation of *or* that can distinguish these accounts.

In classical logic, the meaning of *p or q* is determined by a truth function that maps pairs of truth values, true or false, for each proposition, on to a truth value (Jeffrey, 1981). We use *p* to mean *p* is true and *¬p* (not-*p*) to mean *¬p* is true, and so *p* is false. For any connective, there are four cases to consider. When both are true, *p* and *q,* we use the notation *pq*. An inclusive *or* maps *pq*, *p¬q*, or *¬pq* to true. So, *p or q* is true if and only if *p* and *q* is true (*Anna is an actor and a writer*), or *p* and *¬q* is true (*Anna is an actor and not a writer*), or *¬p* and *q* is true (*Anna is not an actor and a writer*) otherwise *p or q* is false (when *¬p¬q*: Anna *is not an actor and not a writer*). This formulation provides the *disjunctive normal form* (DNF) of *p or q*, which, if true, is equivalent to (*p and q*) *or* (*p and ¬q*) *or* (*¬p and q*). If one of these three disjuncts is true in the actual world, then the disjunction is true. In modal logic (Hughes & Creswell, 1996), if *A* is true, then *possibly A* is true.[1] Therefore, according to standard modal logic, if *p or q* is true, then one of the three disjuncts in the DNF is possible: *possibly*(*p and q*) *or possibly*(*p and ¬q*) *or possibly* (*¬p and q*). If one of the disjuncts is true in the actual world, then there is a possible world in which it is true. As we now see, the mental models theory exploited this implication of classical modal logic in describing the status of mental models (Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002).

Psychological theories of disjunctions include mental models theory, mental logic (Braine & O'Brien, 1998; Rips, 1994), and the "new paradigm" probabilistic approach (Cruz, Over, & Oaksford, 2017). In this paper, we focus on the mental models theory (MMT) of inclusive disjunction, which we just introduced. In MMT1, "Each mental model represents a possibility," and "models represent only the possibilities that are true given a premise" (Johnson-Laird, 2001, p. 434). The mental model of a disjunction is equivalent to the DNF, with modal possibility operators, that we just introduced. However, the actual MMT mental representation dispenses with all the logical operators, so syntactic

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[1] This result follows from the reflexivity axiom *necessarily(A), therefore A*, as follows:*Necessarily(A), therefore A.Necessarily(not-A), therefore not-A.Not-not-A, therefore not-necessarily(not-A).A, therefore possibly(A).*

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markers for *possibly, or*, or *and* are not included (and often the negation tags are omitted).[2] Byrne & Johnson-Laird (2020, p. 761) confirm this interpretation of MMT1, taking a factual disjunction, such as, there is beer, or there is wine or both:

> "An earlier version of the theory [MMT1] proposed that models were disjunctive alternatives—it is possible that there is beer alone, *or* it is possible that there is wine alone, *or* it is possible that there are both."

In MMT1, as in classical logic, *p or q* is true when one of *p*¬*q*, ¬*pq* and *pq* is true; otherwise, it is false (Johnson-Laird & Byrne, 2002, p. 651).

In the recently revised version of MMT, MMT2, "the meanings of compound assertions, such as conditionals (*if*) and disjunctions (*or*), unlike those in logic, refer to sets of epistemic possibilities that hold in default of information to the contrary" (Johnson-Laird & Ragni, 2019; Khemlani et al., 2018). Moreover, "An inclusive disjunction, *A or B or both*, refers by default to the exhaustive conjunction of these default possibilities that each holds if no information is to the contrary: *possible* (A & not B) & *possible*(Not A & B) & *possible*(A & B)" (Khemlani et al., 2018, p. 4). In MMT2, *p or q* is true, if and only if all the three cases are possible and ¬*p*¬*q* is impossible (Hinterecker et al., 2016; Khemlani et al., 2018). This is called the *conjunction of possibilities* (COP) interpretation. The proposal that disjunctions "refer to sets of *epistemic* possibilities" is critical to our experiments. For example, suppose you know that there are no other possibilities for Anna's actual profession, other than writer or actor, then it is not epistemically possible for ¬*p*¬*q* to be true. However, it remains logically possible that she is neither an actor nor a writer, because in some possible imaginary world she is, say, an acrobat.

In summary, the truth table account interprets an inclusive disjunction (*p* or *q*) as the disjunction of the three cases, *pq*, *p*¬*q*, or ¬*pq*, only one of which actually needs to be true for *p* or *q* to be true. MMT1 interprets *p* or *q* as the disjunction of three possibilities, and again it is true if one of the three cases is actually true. MMT2 interprets *p* or *q* as the conjunction of the three cases treated as epistemic possibilities, and it is true only if all three cases are epistemically possible. No previous empirical studies have distinguished between these different accounts, and so we conducted two experiments to resolve this issue.

Both experiments presented participants with the 15 possible combinations of the truth table cases, *p*¬*q*, ¬*pq*, *pq*, and ¬*p*¬*q* (see *Supplementary Online Materials: S2 and S3*). For example, participants may be told that the combinations involve packs of colored cards of different shapes. So, the {¬*pq*, *pq*} pack of cards, say, contains only *non-round red* and *round red* cards. They were then told that a card, for which the description *the card is round or the card is red* is true, was drawn randomly from a pack. However, they are not told the actual identity of the card. Consequently, the pack delineates the epistemic possibilities for the card's identity. In Experiment 1, they were then asked to indicate whether it was possible for it to have been drawn from each of the fifteen packs corresponding to the different combinations of logical cases. In our example, the drawn card could be either *non-round and red* or *round and red*. In either case, according to classical logic and MMT1, the disjunction is true because one of these possibilities must be instantiated for the disjunction to be true. Therefore, it is possible that the card was drawn from this pack. We assume that people are far more likely to say it is possible when the disjunction truly describes the pack, that is, for classical logic and MMT1, all packs that exclude ¬*p*¬*q*, that is,{*p*¬*q*, ¬*pq*, *pq*}, {*p*¬*q*, *pq*}, {¬*pq*, *pq*}, {*p*¬*q*, ¬*pq*}, {¬*pq*}, {*p*¬*q*}, {*pq*} (the seven key sets).

For MMT2, if the disjunction is true of the unknown card, then it must be epistemically possible for the card to be any of *p*¬*q*, ¬*pq*, *pq* but impossible that it is ¬*p*¬*q* (Khemlani et al., 2018, p. 4). In our example set, it is epistemically impossible for the card to be *round and not red*. Consequently, given this {¬*pq*, *pq*} pack of cards, MMT2 must predict that people will judge it to be impossible for the card to have been drawn from this pack because the disjunction could not be true. The only pack where the disjunction is true of the unknown card, is the {*p*¬*q*, ¬*pq*, *pq*} pack (the complete key set) where it is epistemically possible for the card to be any of *p*¬*q*, ¬*pq* and *pq* but not ¬*p*¬*q*. Therefore, for MMT2, people should be more likely to say it is possible that the unknown card was drawn from the complete key set than the remaining 14 packs.

In Experiment 2, participants were asked to judge whether *p or q* is true (e.g., it is *round or red*) of an unknown card randomly drawn from the 15 different packs. This experiment makes similar predictions to Experiment 1. We assume that people will judge this to be a true description only if the card is drawn from a pack in which it is not possible for it to be false. For MMT1 and classical logic, these are the seven key sets, whereas, for MMT2, this is only the case for the complete key set.

One could argue that the question we should be asking in Experiment 1 is from which packs was the card *necessarily* drawn if *p or q* truly describes the card. After all, it is *possible* that a true instance of the disjunction is drawn from any pack apart from {¬*p*¬*q*}. In which case, we should only observe differences between {¬*p*¬*q*} and every other set. However, here we can let the data decide. We have argued that participants will interpret our possibility question to mean the card is only possibly drawn from packs for which it is impossible for the card to be ¬*p*¬*q* and so for the disjunction to be false. Thus, for MMT1, this means all sets that exclude ¬*p*¬*q*. This interpretation is in line with our predictions. Note, however, that whichever interpretation of the possibility question participants adopt, the key distinction between MMT1 and MMT2 remains. According to MMT2, but not MMT1, "possible" judgments should be far higher for {*p*¬*q*, ¬*pq*, *pq*} than the remaining key sets, because according to MMT2, *p or q* is possibly false in any other set, and so is only true in the complete set.

2. Experiments 1 and 2

2.1. Method

Participants. The participants were 188 college students from Shaanxi Normal University in China: 92 participants (39 females) in Experiment 1 and 96 participants (43 females) in Experiment 2. They had not studied logic before. Both experiments received ethical approval from the Shaanxi Normal University Human Research Ethics Committee. All the participants provided written informed consent.

Design and materials. Experiment 1 was a paper-and-pencil questionnaire study using the possibility judgment task with the 15 sets. There were two similar problems with different contents (the card problem and the ball problem), which were between-subjects. In the card problem that is presented in *S2*, participants were told that a card was drawn randomly from a pack of cards and that a disjunction, for example, *the card is round or the card is red* was true of the randomly drawn unknown card. Participants then judged whether each of the 15 sets is possibly the pack of cards from which the card was drawn. In each problem, the 15 sets were presented in counterbalanced order.

Experiment 2 used the truth judgment task with the same materials as Experiment 1. However, for each set in each of the two problems, participants were told that an unknown instance was drawn randomly from the set, and they had to indicate whether the disjunction was true of the drawn instance. We present the example problem as the ball problem in *S3*.

Procedure. Each experiment was conducted in a quiet classroom. Every participant received the questionnaire on a sheet of paper and was given a pen to fill it out. They took about 10 min to complete the questionnaire.

[2] Mental models are supposed to be iconic, providing an image of reality or the possible realities permitted by the truth of a claim, which could not include syntactic operators, although negation tags are used in mental models.

3. Results and discussion

In both experiments, there were no performance differences between the two kinds of problem content. Consequently, the results of the two problems were merged. The overall results are shown in Fig. 1. We analyzed the data (Wang & Zheng, 2020) using Bayesian generalized linear modelling (see, *Supplementary online Materials S1*). We tested all of our predictions by comparing sets using Bayes Factors because our main predictions concern the lack of any difference between the complete key set and the other key sets.

In Experiment 1, we first compared each of the key sets with the corresponding set, including ¬*p*¬*q* (e.g., {*pq*} vs {*pq*, ¬*p*¬*q*}). For all seven comparisons, there was decisive evidence that people regarded the unknown instance as only possibly drawn from one of the key sets ($3.58 \times 10^4 < BF_{10} < 1.92 \times 10^8$) (Kass & Raftery, 1995; see *Supplementary Online Materials: S1*). This result is only consistent with participants interpreting the question to mean the drawn instance was only possibly drawn from sets for which it is impossible for the disjunction to be false. We also found substantial evidence that the impossible set {¬*p*¬*q*} was judged less possible than the other seven sets that include ¬*p*¬*q* ($5.16 < BF_{10} < 3.92 \times 10^4$). However, when we compared the complete key set with each of the six remaining key sets, the evidence for the null was substantial to very strong for five comparisons ($3.01 < BF_{01} < 83.33$) and anecdotal only for {*p*¬*q*} ($BF_{01} = 2.35$). For no comparison was there any evidence that people regarded the remaining key sets as any less possible than the complete key set. These findings are consistent with MMT1 but not MMT2.

In Experiment 2, we carried out the same comparisons as in Experiment 1. For all seven comparisons between the key sets and the corresponding set including ¬*p*¬*q*, there was decisive evidence that people judged *p or q* to be true of the unknown instance only if it was drawn from one of the key sets ($7.87 \times 10^4 < BF_{10} < 5.03 \times 10^9$). We also found mostly substantial or greater evidence that people judged *p or q* true of the unknown instance less for the impossible set {¬*p*¬*q*} than the other seven sets that include ¬*p*¬*q* ($5.70 < BF_{10} < 762.7$), although this was anecdotal for {*p*¬*q*, ¬*p*¬*q*} ($BF_{10} = 2.92$) and {¬*pq*, ¬*p*¬*q*} ($BF_{10} = 2.11$). Comparing the complete key set with each of the six remaining key sets, the evidence for the null was strong or very strong for four comparisons ($16.39 < BF_{01} < 90.91$), although anecdotal for {*p*¬*q*} ($BF_{01} = 2.55$) and {¬*pq*} ($BF_{01} = 1.97$). For no comparison was there any evidence that more participants regarded *p or q* to be true in the complete key set than in the remaining key sets. Again, this finding is consistent with the predictions of the truth table account and MMT1, but not MMT2. Most participants did not regard *p or q* to be true of an unknown instance only when the three cases (*p*¬*q*, ¬*pq*, *pq*) are all epistemically possible, as MMT2 predicts. Participants regarded the disjunction to be true of the unknown instance if one or more these three cases was epistemically possible and ¬*p*¬*q* was epistemically impossible.

4. General discussion

Experiments 1 and 2 tested whether participants read inclusive disjunctions, without the modulation by additional knowledge, as a disjunction or conjunction of possibilities or actualities. The two tasks yielded a parallel response pattern. In both possibility and truth judgments, participants showed the disjunctive interpretation. An inclusive disjunction implies the disjunction of the three possibilities. It is true when one of the three cases must be actual (logic and MMT1). These findings support classical logic and MMT1, but not MMT2. The present results also support the validity of *or*-introduction (*p, therefore, p or q*) (Baratgin et al., 2016; Cruz et al., 2017; Oaksford et al., 2019), which MMT2 regards as invalid (Khemlani et al., 2018).

However, it could be argued that our experimental hypotheses depend on a confusion concerning the meaning of "possible". According to the logical view, any proposition is possible as long as it does not entail a contradiction. This is the logical or *alethic* interpretation of "possible", that we mentioned in the introduction. Clearly, in talking about packs of cards impossibly containing various cases, this is not the interpretation of epistemic possibilities in MMT2. Take the key set {*pq*}, which we can describe by the following list of actual states of affairs (excluding the impossibility ¬*p*¬*q*):

(1) *actually*(*round and red*) & *actually not*(*round and not red*) & *actually not*(*not round and red*).

Alethically, (1) is consistent with the MMT2 account where each case is possible, because whether *A* or ¬*A* is true in the actual world, *possibly A* can hold in some possible imaginary world.

However, as we have observed, MMT2 eschews the alethic interpretation in favor of epistemic modalities, where what is possible is relative to what you know (Hinterecker et al., 2019; Johnson-Laird &

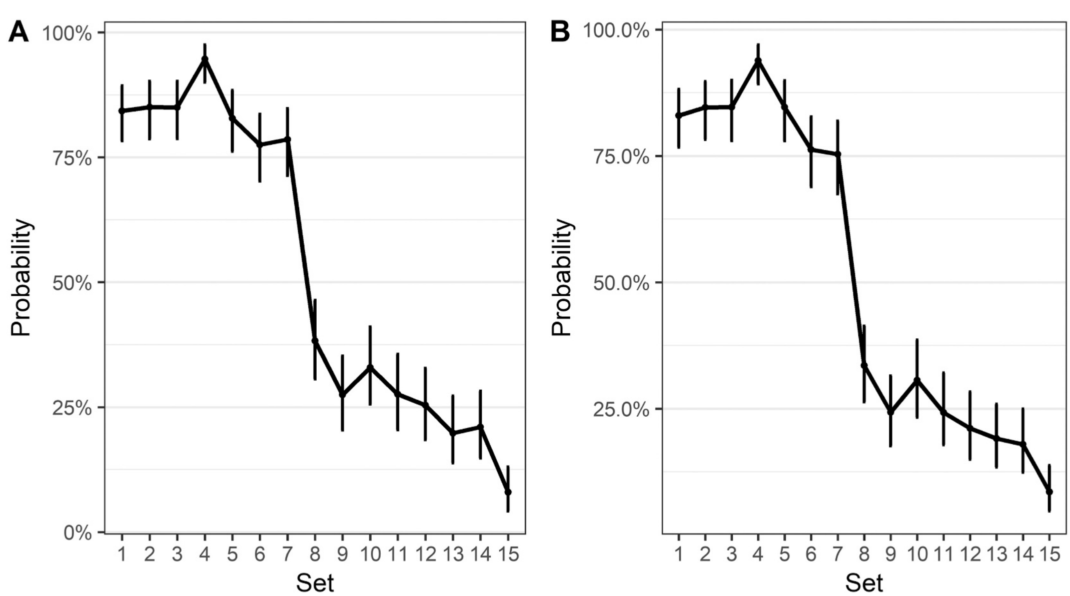

Fig. 1. A: Probability of possibility judgments in Experiment 1; B: Probability of truth judgments in Experiment 2. For sets, the number denotes the following sets: 1 = {*pq*,*p*¬*q*,¬*pq*} (complete key set), 2 = {*pq*,*p*¬*q*}, 3 = {*pq*,¬*pq*}, 4 = {*pq*}, 5 = {*p*¬*q*,¬*pq*}, 6 = {*p*¬*q*}, 7 = {¬*pq*} (2–7 = incomplete key sets), 8 = {*pq*,*p*¬*q*,¬*pq*,¬*p*¬*q*}, 9 = {*pq*,*p*¬*q*,¬*p*¬*q*}, 10 = {*pq*,¬*pq*,¬*p*¬*q*}, 11 = {*p*¬*q*,¬*pq*,¬*p*¬*q*}, 12 = {*pq*,¬*p*¬*q*}, 13 = {*p*¬*q*,¬*p*¬*q*}, 14 = {¬*pq*,¬*p*¬*q*}, 15 = {¬*p*¬*q*}. Error bars = 95% HDI.

Ragni, 2019). MMT2 has not provided a theory of epistemic modality, but according to the account proposed by Oaksford et al. (2019), *p* is possible as long as nothing you know rules it out, that is, you cannot prove ¬*A* from what you know. Knowing that, for example, the pack does not contain *round and not red* cards, means you can infer ¬*contains* (*round and not red*) from what you know and so, epistemically, *round and not red* is impossible. That is, epistemically, *actually not containing* (*round and not red*) entails *impossibly*(*round and not red*). So, the epistemic modal reading of (1) is:

(2) *possibly*(*round and red*) *&* *impossibly*(*round and not red*) *&* *impossibly*(*not round and red*).

This statement (2) is not consistent with the conjunctive interpretation of MMT2. Consequently, according to the epistemic interpretation of these modal terms, argued for explicitly by MMT, our findings are only consistent with the disjunctive interpretation of MMT1.

It could be argued that these experiments do not support the standard logical interpretation of disjunction, which is extensional. Whether *p* or *q* truly describes the instance drawn from the pack depends solely on the actual identity of the card. Whether, with respect to a pack, it *must* (epistemically) be one of the cases that make the disjunction true is irrelevant from this perspective. Nevertheless, the disjunctive DNF interpretation, where the disjunction is true in all the key sets, is consistent with standard logic in a way that the conjunctive COP interpretation, where the disjunction is true only in the complete key set, is not.

In conclusion, these results could be regarded as unremarkable in that they are consistent with the standard logical interpretation of disjunction in classical logic and new paradigm probabilistic approaches. However, they disconfirm the claims of MMT2 to provide a radically different empirical interpretation of the disjunction. These results cast serious doubt on the conjunctive interpretation of MMT2 but are consistent with the previous theoretical arguments that the conjunctive interpretation is logically untenable (Baratgin et al., 2015; Cruz et al., 2017; Oaksford et al., 2019). Our experiments substantiate these theoretical arguments and suggest that the revised mental model theory is empirically, as well as theoretically problematic.

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