Consistency and inconsistency between instance and categorical modal inferences from conditionals

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Abstract

For the modal meanings of conditionals with the form *if p then q*, existing psychological accounts imply different modal meanings of *if p then q*. For instance and categorical modal inferences from conditionals in sampling problems, the basic principle is that whether or not a set includes some case determines whether or not an instance randomly drawn from the set is possibly this case. One experiment investigated the relationship between instance and categorical modal inferences from conditionals. The results demonstrate that (1) cases *pq* and *p¬q* tend to elicit consistent response patterns conforming to the basic principle, but cases *¬pq* and *¬p¬q* tend to elicit inconsistent response patterns violating the basic principle; (2) the inconsistent response pattern of not-*p* cases can be explained by the pragmatic heuristic that no evidence can rule out that not-*p* cases are possible; (3) the overall pattern of modal inferences is beyond each existing theory. Alternatively, we propose a revised suppositional theory (ST3) integrating the constraint of the truth conditions of conditionals in the Jeffrey table and the pragmatic heuristic. It can explain the overall pattern of modal inferences. Overall, a true conditional requires that *pq* is possible and *p¬q* is impossible for the instance of the true conditional, but not that not-*p* cases are possible. This favors the pragmatic heuristic account with ST3 over the semantic compatibility assumption in mental models theories.

Keywords: conditionals; modal inferences; inconsistency; the Jeffrey table; the pragmatic heuristic

Introduction

In the psychology of propositional reasoning, a current question is how people understand the modal meanings of basic conditionals with the form *if p then q* (Hinterecker, Knauff, & Johnson-Laird, 2016; Khemlani, Byrne, & Johnson-Laird, 2018; Schroyens, 2010; Wang, Yin, & Zheng, 2018). For basic conditionals there is no specific background knowledge that modulates the relationship between antecedents and consequents (Evans & Over, 2004; Johnson-Laird & Byrne, 2002). In this paper, we focus on singular conditionals referring to singular objects/instances (e. g., *if the card is round then it is red*). We illustrate the modal question as the following sampling problem. For an unknown card randomly drawn from a pack of cards, given that it is true of the card that *if the card is round then it is red*, what case is possible or impossible for the card? And what case does the pack of cards necessarily, possibly or impossibly includes? We call the former the instance modal question in that it is about modal inferences for the particular object/instance referred to by the conditional. We call the latter the categorical modal question in that it is about modal inferences for the set/category of objects/instances from which the particular object/instance referred to by the conditional was drawn.

For the two modal questions in the above sampling problem, the true conditional constrains instance inferences for the instance modal question, and instance inferences further constrains categorical inferences for the categorical modal question. Logically, categorical inferences should be consistent with instance inferences, which should in turn be consistent with the interpretation of the true conditional. In particular, the interpretation of the true conditional determines whether the card referred to by the conditional is possibly each of four cases (a round red card, a round non-red card, a non-round red card, and a non-round non-red card). For example, given the true conditional, when the card is possibly a non-round red card of the four cases, the pack must include non-round red cards, because if the pack does not include such cases then the card from the pack is impossibly the non-round red card, as contradicts that the card is possibly the non-round red card. In other words, the card from the pack may be the non-round red card only if the pack includes non-round red cards. Here, we obtain Rule 1: the set from which the instance of a true conditional was randomly drawn, must include any case that is judged possible for the instance. When the card is impossibly a round non-red card, the pack cannot include round non-red cards, because if the pack includes such cases then the card is possibly the round non-red card, as contradicts that the card is impossibly the round non-red card. In other words, the card from the pack cannot be the non-round red card only if the pack excludes non-round red cards. Here, we obtain Rule 2: the set from which the instance of a true conditional was randomly drawn, cannot include any case that is judged impossible for the instance. Moreover, when it is uncertain whether the card is possibly a non-round red card or not, the card is possibly the non-round red card or not. When the card is possibly the non-round red card, the pack must include non-round red cards, according to Rule 1. When the card is impossibly the non-round red card, the pack must exclude non-round red cards, according to Rule 2. Thus, the uncertain instance inference implies that the pack may include (that is, may exclude) non-round red cards. Here, we obtain Rule 3: the set from which the instance of a true conditional was randomly drawn, may include any case that is judged uncertain for the instance. Here, epistemically "may include" is equivalent to "may exclude", according to the principle of presuppositions where *It is possible that A* presupposes *It is possible that not-A* (Ragni & Johnson-Laird, 2020). Thus, "may include" means both "may include" and "may exclude".

Overall, for the sampling problem, the three categorical inference ("must include", "cannot include", and "may include") are consistent with the three instance inferences ("may be", "cannot be", and "uncertain"), respectively. Any other correspondence between category and instance inferences are inconsistent. For example, "may include" is inconsistent with "may be" and "cannot be". The above three rules of solving the sampling problem are based on the basic sampling principle that whether or not a set includes some case determines whether or not an instance that was randomly drawn from the set is possibly this case. This principle is really and epistemically valid because the instance is one element of the set and so whether it is possible or impossible is determined by the distribution of cases in the set. No previous studies have tested the three rules.

Previous studies focused on instance inferences from conditionals, but not categorical inferences from conditionals (Barrouillet, Gauffroy, & Lecas, 2008; Barrouillet, Grosset, & Lecas, 2000; Goodwin, 2014; Handley, Evans & Thompson, 2006; Schroyens, 2010; Wang, Yin, & Zheng, 2018). These studies found that given a true singular conditional with the form *if p then q*, people generally judge each of three cases (*pq*, ¬*pq*, ¬*p*¬*q*) possible for the instance referred to by the conditional, but *p*¬*q* impossible. According to these findings and the three rules of modal inferences, we predict that each of the three cases is possible for the instance referred to by the conditional and so must be present in the set from which the instance was randomly drawn, but *p*¬*q* is impossible for the instance and so it cannot be present in the set.

Existing psychological accounts for the meanings of basic conditionals imply different predictions for the two questions and the relationship between instance and categorical inferences. Thus, investigating the relationship can distinguish between existing different accounts. However, no previous studies have investigated the relationship and tested the three rules. Current research aims to investigate the relationship and to test the existing different accounts for basic conditionals.

Next, we specify the complete predictions for instance and categorical inferences of existing main psychological accounts for the modal meanings of conditionals. The main psychological accounts are mental models theories (Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002;

Khemlani, Byrne, & Johnson-Laird, 2018), suppositional theories (Evans & Over, 2004; Evans, Over, & Handley, 2005; Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011; Over & Evans, 2003), and the integration theory (Barrouillet, Gauffroy, & Lecas, 2008). Mental logic accounts for conditional reasoning do not address the modal meanings of conditionals (Rips, 1994). The truth table account (shown in Table 1) of classical logic and the hypothetical inference theory focuses on the truth conditions of conditionals, but not the modal meanings of conditionals (Douven, et al., 2018; Jeffrey, 1981; Skovgaard-Olsen et al., 2016).

Table 1

Truth tables for singular conditionals

Cases	*if p then q*		
	Classical truth table	de Finetti table	Jeffrey table
p & q	True	True	True
p & ¬q	False	False	False
¬p & q	True	Void/uncertain	$P(q\|p)$
¬p & ¬q	True	Void/uncertain	$P(q\|p)$

Note: ¬ = not, and $P(q|p)$ = the conditional subjective probability of *q* given *p*.

The mental models theory postulates that the meanings of compound assertions, such as conditionals (*if*) and disjunctions (*or*), unlike those in logic, refer to sets of epistemic possibilities (that is, mental models) that hold in default of information to the contrary (Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002; Khemlani, Byrne, & Johnson-Laird, 2018; Johnson-Laird & Ragni, 2019). Mental models allowed by a given assertion are possibilities that people can envisage, which are possible states of affairs semantically compatible with the given assertion. In Johnson-Laird &

Byrne (2002), the core meaning of *If A then C* refers to the possibilities:

$a \quad c$

$\neg a \quad c$

$\neg a \quad \neg c$

In the model theory, the true conditional does not tolerate exceptions (*a* & ¬*c* cases), that is, it is deterministic. Thus, *a* & ¬*c* cases are impossible for the true conditional (Goodwin, 2014).

The model theory has the previous version (MMT1) (Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002) and the revised version (MMT2) (Hinterecker, Knauff, & Johnson-Laird, 2016; Johnson-Laird, Khemlani, & Goodwin, 2015; Khemlani, Byrne, & Johnson-Laird, 2018). Next, we specify the differences between MMT1 and MMT2, which are quoted as follows:

"The previous version of the model theory [MMT1] postulated that people understand compound assertions by constructing models representing the disjunctive alternatives to which they refer. The new theory [MMT2] postulates instead that compounds refer to conjunctions of possibilities that each hold in default of information to the contrary" (Khemlani, Byrne, & Johnson-Laird, 2018, p. 4). "In the earlier version of the model theory, the possibilities to which an assertion referred were in a disjunction; in the present version, the possibilities hold in default of information to the contrary, and they are in a conjunction" (Khemlani, Byrne, & Johnson-Laird, 2018, p. 5).

According to the above two quotations, for the instance modal question, in MMT1 the conditional *if p then q* means that *pq* is possible or ¬*pq* is possible or ¬*p*¬*q* is possible, but in MMT2 the conditional means that *pq* is possible and ¬*pq* is possible and ¬*p*¬*q* is possible. In both versions, each of *pq*, ¬*pq*, and ¬*p*¬*q* is possible, according to the compatibility assumption in MMT that a case semantically compatible with a given assertion is possible for the assertion (Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002). The difference between both versions is that the three possibilities are in the disjunction in MMT1, but in the conjunction in MMT2.

In MMT1, for the instance modal question, *if p then q* implies that the instance may be *pq* or ¬*pq* or ¬*p*¬*q*, but is impossibly *p*¬*q*. Thus, which of the three possible cases is the instance is uncertain, and a true conditional does not require each of the three possible cases. This point is confirmed by Goodwin and Johnson-Laird (2018). They claimed "any case which an assertion refers to as possible should also be one in which the assertion is true." And they argued that a conditional should hold in each possible case allowed by the conditional. They used the "collective" truth task that had participants to judge whether a set of three assertions about an instance, such as: *If A then C*, *not-A*, *C*, could all be true at the same time. They predicted that all the three assertions could hold in *not-A* & *C*. Likewise, all three assertions (*If A then C*, *not-A*, *not-C*) could hold in *not-A* & *not-C*. And all three assertions (*If A then C*, *A*, *C*) could hold in *A* & *C*. The results are consistent with these predictions. Overall, the conditional could be true in *A* & *C* or *not-A* & *C* or *not-A* & *not-C*. Thus, when the conditional is true, the instance of the conditional may be *A* & *C* or *not-A* & *C* or *not-A* & *not-C*. Here, each of the three cases is possible for the instance, but each of the three possible cases is not required by the true conditional. This is in accordance with the interpretation of the disjunction of possibilities in MMT1.

Each of *pq*, ¬*pq*, and ¬*p*¬*q* is possible for the instance of the true conditional *if p then q*, and so it must be present in the corresponding set according to Rule 1. This categorical inference is inconsistent with the following categorical inference prediction by MMT1. According to the interpretation of the disjunction of possibilities in MMT1, each of the three possible cases is not required by the true conditional and so it may be absent in the corresponding set. The two categorical inferences are not incompatible. Thus, MMT1 is not self-consistent in terms of categorical inferences. It is unable to predict categorical inferences of the three possible cases. It predicts that $p\neg q$ is possible for the instance and so must be absent in the set according to Rule 2.

In MMT2, for the instance modal question, *if p then q* implies that pq, $\neg pq$ and $\neg p\neg q$ are all possible for the instance referred to by the conditional, but $p\neg q$ is impossible. Thus, for the categorical modal question, *if p then q* implies that pq, $\neg pq$ and $\neg p\neg q$ must all be present in the set from which the instance is randomly drawn (according to Rule 1), but $p\neg q$ cannot be present in the set (according to Rule 2). Thus, a true conditional requires that the set must include each of the three cases, but cannot include $p\neg q$ cases. Overall, in MMT2 each of the three possible cases is required by the true conditional.

The suppositional theory (ST) explains the meanings of conditionals (*if p then q*) as hypothetical test (Evans, Ellis, & Newstead, 1996; Evans & Over, 2004; Evans, Over, & Handley, 2005; Fugard, et al., 2011; Handley, Evans, & Thompson, 2006; Over & Evans, 2003; Politzer, Over, & Baratgin, 2010). A conditional (*if p, then q*) expresses people's subjective degree of belief in a conditional that relies on the *Ramsey test* in which people hypothetically add the antecedent to their stock of knowledge and judge whether the consequent holds. People tend to judge a conditional true/acceptable/believable in the high subjective conditional probability of the consequent given the antecedent, $P(q|p)$. In ST, "true" may be semantical or pragmatic, depending on contexts. A subjective conditional probability may be based on an available objective conditional probability or some subjective pragmatic factors (Over, 2020). Accordingly, people interpret a true conditional as highly probable/believable rather than logically valid or necessary (Adams, 1998; de Finetti, 1995; Evans & Over, 2004; Evans, 2007; Jeffrey, 1991; Kleiter, Fugard, & Pfeifer, 2018; Liu, Lo, & Wu, 1996; Oaksford & Chater, 2001, 2009; Over, 2016; Over & Cruz, 2017; Over et al., 2007; Singmann, Klauer, & Over, 2014; Skovgaard-Olsen, Singmann, & Klauer, 2016; Wang & Yao, 2018; Wang & Zhu, 2019). In the *Ramsey* test, only pq and $p\neg q$ cases affect $P(q|p)$, and so they are relevant to the truth or falsity of a conditional. Only *pq* cases are required by a true conditional. In contrast, ¬*pq* and ¬*p*¬*q* cases do not affect $P(q|p)$, and so they are irrelevant to and can not determine the truth or falsity of the conditional. They are unrequired by the true conditional. This is the irrelevance assumption of not-*p* cases that has been confirmed by the de Finetti truth table (that is, the defective truth table) with "irrelevant" or "uncertain" assigned to not-*p* cases (see Table 1) (Baratgin, Over & Politzer, 2013, 2014; Barrouillet, Gauffroy, & Lecas, 2008; Evans & Over, 2004; Over & Baratgin, 2017). We use ST1 to denote the suppositional account based on the de Finetti truth table as the defective truth table, in which not-*p* cases neither make the conditional true nor false and so the truth or falsity of the conditional is void or uncertain in not-*p* cases.

ST1 predicts the modal meanings of the suppositional conditional as follow. For the instance modal question, the true conditional implies that *pq* is possible for the instance, and irrelevant ¬*pq* and ¬*p*¬*q* are also possible, but *p*¬*q* is impossible (Handley, Evans, & Thompson, 2006, p 565). Consequently, for the category modal question, the true conditional implies that the set from which the instance was randomly drawn necessarily includes *pq*, ¬*pq* and ¬*p*¬*q* cases (according to Rule 1), but impossibly includes *p*¬*q* cases (according to Rule 2). Thus, ST1 has the same predictions as MMT2. The categorical inference that ¬*pq* and ¬*p*¬*q* cases must be present in the set is inconsistent with the prediction of the irrelevance assumption of not-*p* cases that irrelevant not-*p* cases are not required by the true conditional and so may be absent in the set. Thus, ST1 is not logically self-consistent in terms of categorical inferences of not-*p* cases. It is unable to predict categorical inferences of not-*p* cases.

Moreover, there is the second version (ST2) of ST that is based on the Jeffrey table in Table 1. The Jeffrey table is based on the Ramsey test, according to which people evaluate *if p then q* in ¬*p* cases by hypothetically supposing *p*, making any changes necessary to preserve consistency, and then judging the extent to which *q* follows, with a judgment about $P(q|p)$ as a result (Adams, 1998; Edgington, 2010; Jeffrey, 1991; Over & Baratgin, 2016; Pfeifer & Kleiter, 2010). This is the conditional probability hypothesis that probability judgments of the conditional are based on $P(q|p)$. In the Jeffrey table of ST2, in not-*p* cases the conditional can be judged true or false, depending on whether or not available $P(q|p)$ is high in context (Over & Cruz, 2018; Wang & Zhu, 2019). In ST2, not-*p* cases are still irrelevant to the truth or falsity of the conditional, but available $P(q|p)$ in not-*p* cases can determine the truth or falsity of the conditional.

ST2 does not make predictions for the modal implications of conditionals. Next, we logically derive the modal implications of conditionals from the truth conditions of conditionals in ST2 as follow. In the Jeffrey table of ST2, *pq* is the case that makes the conditional *(if p then q*) true, *p¬q* is the case that makes the conditional false, and in not-*p* cases the conditional can be judged true when $P(q|p)$ is high. Thus, in the present sampling task, the conditional is true of the instance from the set <u>only in</u> either the situation where the instance is a *pq* case, or the situation where the instance is a not-*p* case in the set where $P(q|p)$ is high and so *pq* cases must be present. Thus, in ST2, that the conditional is true of the instance implies that the instance is either *pq* or not-*p*, but cannot be *p¬q*. This implies that either *pq* or not-*p* is possible for the instance but *p¬q* is impossible. Thus, each of *pq* and not-*p* may be possible or impossible and so it is uncertain whether each of *pq* and not-*p* is possible or impossible for the instance. Consequently, it is also uncertain whether each of *¬pq* and *¬p¬q* is possible or impossible for the instance. Overall, the instance of the true conditional may be *pq* or *¬pq* or *¬p¬q*, but cannot be *p¬q*. For categorical inferences, there are only the two situations that make the conditional true and in each situation there must be *pq* cases in the set. Thus, the set must include *pq* cases required by the true conditional. According to Rule 3, the uncertain instance inference that each of *¬pq* and *¬p¬q* is possible or impossible implies the categorical inference that the set may include or exclude each of both. Due to the irrelevance assumption of not-*p* cases, it is reasonable that ¬*pq* and ¬*p*¬*q* cases may be absent or present in the set. According to Rule 2, the instance inference that the instance cannot be *p*¬*q* implies the categorical inference that the set cannot include *p*¬*q* cases. Overall, in ST2 the true conditional imply that the set must include *pq* cases, and may include not-*p* cases, but cannot include *p*¬*q* cases. In ST2, categorical inferences are consistent with instance inferences, and so ST2 is logically self-consistent. Overall, in ST2, instance and categorical modal inferences are based on the constraint of the truth conditions of conditionals in the Jeffrey table, and so only *pq* cases is required by the true conditional.

Moreover, Barrouillet, Gauffroy, & Lecas (2008) proposed an integration theory (IT) that integrates the defective truth table of ST1 and the original mental model account. In the integration theory, truth judgments of conditionals conform to the de Finetti truth table, but modal implications of conditionals conform to the compatibility assumption in MMT. In this theory, only the truth condition (*pq*) of the conditional corresponds to the initial model of the conditional, whereas irrelevant cases ¬*pq* and ¬*p*¬*q* correspond to the two implicit models of the conditional. In the present sampling task, modal inferences from conditionals should be based on the compatibility assumption in MMT. In particular, *pq,* ¬*pq,* and ¬*p*¬*q* are all semantically compatible with *if p then q* and so all are possible for the instance, but *p*¬*q* is semantically incompatible with the conditional and so is impossible for the instance. According to Rule 1, each of the three possible cases must be present in the corresponding set. According to Rule 2, the impossible case *p*¬*q* must be absent in the corresponding set. Overall, the set must include *pq,* ¬*pq,* and ¬*p*¬*q* cases, but cannot include *p*¬*q*. Thus, the integration theory has the same modal implications of conditionals as MMT2.

Moreover, in modal logic, Stalnaker (1968) proposed a possible world theory (PWT) of conditionals. Its core idea is as follows: “Consider a possible world in which *A* is true, and which otherwise differs minimally from the actual world. *"If A, then B"* is *true (false) just in case B* is *true (false) in that possible world*" (Stalnaker,1968, p. 45). Here, the conditional is true only if there is a possible world (being maximally similar to the actual world) where *B* is true given that *A* is true, regardless of whether *A* is true or false in the actual world. A true conditional *if p then q* is based on the differentiation between the actual world and the possible world. PWT is unable to make predictions for the present sampling task, because in such a task a conditional is true of an instance from an unknown set and so there is not the differentiation between the actual world and the possible world. Thus, PWT is not applicable to the present sampling task.

Table 2 The main theoretical predictions of modal inferences from conditionals (*if p then q*)

Accounts	Instance inferences	Categorical inferences
MMT1	The instance may be *pq* or ¬*pq* or ¬*p*¬*q*, but cannot be *p*¬*q*.	The set cannot include *p*¬*q*.
MMT2 / IT	*pq*, ¬*pq*, and ¬*p*¬*q* are all possible. *p*¬*q* is impossible.	The set must include *pq*, ¬*pq*, and ¬*p*¬*q*, but cannot include *p*¬*q*.
ST1	*pq*, ¬*pq*, and ¬*p*¬*q* are possible. *p*¬*q* is impossible.	The set must include *pq*, but cannot include *p*¬*q*.
ST2	*pq* or ¬*pq* or ¬*p*¬*q* is possible. Whether each of the three case is possible is uncertain. *p*¬*q* is impossible.	The set must include *pq*, may include ¬*pq* and ¬*p*¬*q*, but cannot include *p*¬*q*.

In summary, MMT2 /IT, ST1, and ST2 make different predictions for instance and categorical modal inferences from basic conditionals with abstract contexts. The complete predictions of these theories are shown in Table 2. The key difference between their predictions is which of the three possible cases (*pq*, ¬*pq*, and ¬*p*¬*q*) compatible with a true conditional is required by it. In MMT2 and IT, each possibility is required by the true conditional, and so each possible case must be present in the set. In MMT1, each possibility is not required by the true conditional, and so each possible case need not be present in the set. In ST1 and ST2, only the possibility of *pq* is required by the true conditional and so only *pq* cases must be present in the set. We conducted one experiment to test the predictions in Table 2 and investigate the relationship between instance and categorical modal inferences of conditionals.

Experiment

Method

Participants

The experiment used a between-group factor design with two groups of participants, a power analysis of this factor indicated that 62 participants in total would be required to detect an effect size of .4 with a power of 90%. We aimed for 80 participants in each group and so there were 160 participants in total. 160 college students (67 men, 93 women) from Shaanxi Normal University in China participated in the experiment. They had not studied any logic course before. They were between 18 and 21 years old. The experiment received ethical approval from the University Human Research Ethics Committee, and all of the participants signed a written consent form.

Design and materials

The experiment was a paper-pen questionnaire study. In order to test whether categorical modal inferences would be consistent with instance modal inferences, we used a between-subjects design where order of questions was varied with two orders, each of which was arranged to one group of participants. In one order, 80 participants answered an instance modal question followed by the categorical modal question, as presented in the following questionnaire. In the other order, 80 participants answered a categorical modal question followed by the instance modal question. The two orders were counterbalanced. This design aimed to examine whether or not order of questions would affect instance/categorical modal inferences and so whether or not there would be some interaction between preceding instance inferences and subsequent categorical inferences. In each order, there were two between-subjects problems with different materials. One was the card problem with the conditional *if the card is round then it is red* is presented as follows. It is the English version of the questionnaire, translated from the original Chinese. The other was the ball problem with the conditional *if the ball is blue then it is plastic* had the same format of questions as the card problem. Each problem was completed by 40 participants.

Instruction

Please complete the following problem by the presentation order of items. Please tick your answers.

There is a pack of cards. A card was randomly drawn from the pack of cards. The statement “if the card is round then it is red” is true for the card.

1. Please judge whether the card is possibly the respective cases below.

A. A non-round non-red card (is possibly is impossibly uncertain)

B. A non-round red card (is possibly is impossibly uncertain)

C. A round non-red card (is possibly is impossibly uncertain)

D. A round red card (is possibly is impossibly uncertain)

2. Please judge whether the pack of cards necessarily includes the respective cases below.

A. Non-round non-red cards (necessarily includes possibly includes impossibly includes)

B. Non-round red cards (necessarily includes possibly includes impossibly includes)

C. Round non-red cards (necessarily includes possibly includes impossibly includes)

D. Round red cards (necessarily includes possibly includes impossibly includes)

The above instance modal question involved the "uncertain" option, and so can test the "uncertain" predictions in ST2. The instance modal question was similar to the individual possibility judgement question in the previous studies that asked participants to judge whether each of the four cases is individually possible or impossible for the instance referred to by a true conditional. In such questions, each of the four cases may individually be judged possible or impossible. Such questions are unable to determine whether several possible cases are disjunctive or conjunctive and whether each possible case is required by the true conditional. Thus, such questions are unable to distinguish between the different predictions for instance inferences. In the categorical modal question, the inference that the set "necessarily includes" some case implies that this case must be possible for the instance that was randomly drawn from the set, and the possibility of this case is required by the true conditional. The inference that the set "possibly includes" some case implies that the set may include or exclude this case, and so this case may be possible or impossible for the instance, and so the possibility of this case is not required by the true conditional. Thus, categorical modal questions are able to distinguish between the different predictions of the five theories in Table 2. The present sampling task involved both the instance modal question and the categorical modal question, and so was able to distinguish between the different predictions of the five theories.

Procedure

In quiet classrooms, participants were randomly arranged to one order of questions, and they took about 5 minutes to complete the questionnaires.

Transparency and openness

We report how we determined our sample size, all manipulations, and all measures in the study, All materials are available in the section of *Design and materials*. All data for the study are available on OSF (https://osf.io/9td6w/). This study's design and its analysis were not pre-registered.

Results

In each order, the results of the two kinds of problem materials were collapsed across the card and ball problem. The results of the two experimental conditions are shown in Table 3.

For each of the four cases, we conducted a 2 × 2 chi-square test of independence for the difference between the two orders of questions where responses were classed into two categories: the dominant response and the other responses. It revealed that the two orders of questions showed no significant differences in performance. Thus, order of questions made no differences to instance and categorical modal inferences of each case. In each order, previous instance inferences (or categorical inferences) did not affect subsequent categorical inferences (or instance inferences).

Table 3

The results (percentages) in the two experimental conditions

	Instance inferences prior to categorical inferences						Categorical inferences prior to instance inferences					
	Instance inferences			Categorical inferences			Categorical inferences			Instance inferences		
	Imp	Un	Pos	ImpI	PosI	NecI	ImpI	PosI	NecI	Imp	Un	Pos
pq	1	1	98	0	36	64	1	26	73	1	1	98
p¬*q*	92	4	4	79	16	5	84	14	2	98	1	1
¬*pq*	10	11	79	4	91	5	1	89	10	0	16	84
¬*p*¬*q*	4	11	85	1	89	10	1	86	13	2	14	84

Notes: Imp, Un, and Pos denote "is impossibly", "uncertain", and "is possibly", respectively. ImpI, PosI, and NecI denote "Impossibly includes", "possibly includes", and "necessarily includes", respectively.

Table 4

The final merged results (percentages) in the experiment

	Instance inferences			Categorical inferences		
	Is impossibly	Uncertain	Is possibly	Impossibly includes	Possibly includes	Necessarily includes
pq	1	1	98	1	31	68
p¬q	95	2	3	81	15	4
¬pq	5	14	81	2	90	8
¬p¬q	3	13	84	1	88	11

Thus, the results were collapsed across the two orders. The final merged results are shown in Table 4. The response pattern of each order of questions was similar to the overall response pattern in Table 4. For instance inferences, most participants judged each of the three cases (*pq*, *¬pq*, and *¬p¬q*) possible for the instance of a conditional, but *p¬q* impossible; for each case, a binomial test of the difference between the dominant response and the sum of the other two responses showed $p = .000 < .001$. The pattern of instance inferences is consistent with only the common prediction of MMT1, MMT2, IT, and ST1. For categorical inferences, most participants judged that the set necessarily includes *pq* cases, and possibly includes *¬pq* and *¬p¬q* cases, but impossibly includes *p¬q* cases; for each case, a binomial test of the difference between the dominant response and the sum of the other two responses showed $p = .000 < .001$. The pattern of categorical inferences is consistent with only the prediction of ST2. In Table 4 the overall pattern of instance and categorical inferences is beyond the prediction of each existing theory. Moreover, the patterns of instance and categorical inferences were both reliable because order of questions made no differences to instance and categorical inferences of each cases. Consequently, the overall pattern of modal inferences was also reliable.

Next, we examined whether categorical inferences were consistent with instance inferences. Table 5 shows the percentages of the nine combinatory response patterns of the three kinds of instance inferences and the three kinds of categorical inferences for each case. The dominant responses to the four cases (*pq*, *p*¬*q*, ¬*pq*, and ¬*p*¬*q*) were PN (66%), II (80%), PP (78%), and PP (77%), respectively. For each case, the rate of dominant responses was significantly higher than the rate of the sum of the other responses (a binomial test showed p = .000 <.001). For *pq*, most participants judged both that the instance of a conditional is possibly *pq* and that the set including the instance necessarily includes *pq* cases, showing the consistent response pattern predicted by Rule 1. This result is consistent with the common prediction of MMT2, IT, ST1, and ST2 that *pq* cases are required by the conditional and so the corresponding set necessarily includes *pq* cases. For *p*¬*q*, most participants judged both that the instance of a conditional is impossibly *p*¬*q* and that the corresponding set impossibly includes *p*¬*q* cases, showing the consistent response pattern predicted by Rule 2. This result is consistent with the common prediction of all the five theories in Table 2. For ¬*pq* / ¬*p*¬*q*, most participants judged both that the instance of a conditional is possibly ¬*pq* / ¬*p*¬*q* and that the corresponding set possibly includes ¬*pq* / ¬*p*¬*q* cases, showing the inconsistent response pattern. This inconsistency violates Rules 1 and 3. It is beyond each theory. Overall, relevant cases *pq* and *p*¬*q* tended to elicit consistent response patterns, but irrelevant cases ¬*pq* and ¬*p*¬*q* tended to elicit inconsistent response patterns violating Rules 1 and 3. Thus, whether or not participants showed consistent response patterns depended on whether type of cases was relevant or irrelevant. The instance inference that some case is possible for the instance does not always corresponds to the consistent categorical inference that the set must include such cases.

Table 5

Response patterns (percentages) in the experiment

Instance inferences	Categorical inferences	response patterns	*pq*	*p*¬*q*	¬*pq*	¬*p*¬*q*
	Necessarily includes	PN (consistent)	66	0	2	7
Is possibly	Possibly includes	PP (inconsistent)	31	1	78	77
	Impossibly includes	PI (inconsistent)	1	1	1	1
	Necessarily includes	IN (inconsistent)	1	3	1	1
Is impossibly	Possibly includes	IP (inconsistent)	0	13	3	2
	Impossibly includes	II (consistent)	0	80	1	1
	Necessarily includes	UN (inconsistent)	1	1	5	3
Uncertain	Possibly includes	UP (consistent)	0	1	8	8
	Impossibly includes	UI (inconsistent)	0	0	1	0

Notes: In each response pattern, the first capital letter indicates the corresponding instance inference, and the second capital letter indicates the corresponding categorical inference. In the brackets, "consistent" and "inconsistent" represent whether categorical inferences were consistent or inconsistent with instance inferences.

Discussion

In the experiment, order of questions made no differences to instance and categorical inferences of each of the four cases. Most participants showed the inconsistent response pattern of not-*p* cases. Thus, instance and categorical inferences of not-*p* cases were independent of each other. The overall response pattern of instance and categorical inferences is beyond the prediction of each theory. In particular, instance inferences generally conformed to the prediction of MMT/IT/ST1, whereas categorical inferences generally conformed to only the constraint of the truth conditions of conditionals in ST2. For a sampling problem, the truth conditions of conditionals in ST2 implies that only *pq* cases are required by the true conditional and so they must be present in the set, whereas irrelevant cases ¬*pq* and ¬*p*¬*q* are not required by the true conditional and so they may be absent or present in the set.

In the inconsistent response pattern of not-*p* cases, people judge that the instance of the true conditional may be not-*p* cases that may be present or absent in the corresponding set. According to Rule 1, the set must include not-*p* cases that are possible for the instance. According to Rule 3, for not-*p* cases that may be present or absent in the set, whether they are possible or impossible for the instance is uncertain. Thus, the inconsistent response pattern of not-*p* cases violates Rules 1 and 3, and so it is logically irrational. And the inference that the instance may be not-*p* cases is logically incorrect. The correct answer should be that whether the instance is possibly or impossibly a not-*p* case is uncertain. Not-*p* cases may be possible or impossible for the instance, depending on whether they are present or absent in the set. This suggests that the compatibility assumption of not-*p* cases is logically untenable. Judging not-*p* cases possible for the instance should be due to the pragmatic heuristic that in abstract contexts no available evidence can rule out that not-*p* cases are possible for the instance, as is suggested the heuristic that given an inclusive disjunction, *p or q*, people generally judge each of *pq*, ¬*pq*, and *p*¬*q* possible, due to that no available evidence can rule out that these cases are possible for the disjunction (Oaksford, Over, & Cruz, 2019; Wang & Zheng, 2021). The pragmatic heuristic is supported by the finding that relevant cases *pq* and *p*¬*q* tended to elicit consistent response patterns, whereas irrelevant cases ¬*pq* and ¬*p*¬*q* tended to elicit inconsistent response patterns. In particular, *pq* as evidence for the true conditional is required by it, but *p*¬*q* as counterevidence for it is ruled out by it. Thus, the true conditional implies that the set must include *pq* cases but cannot include *p*¬*q* cases. Here, the true conditional can determine whether relevant cases *pq* and *p*¬*q* cases are present or absent in the set. In contrast, not-*p* cases are irrelevant to the true conditional, and no available evidence can rule out irrelevant not-*p* cases in the set, unlike that the true conditional can rule out *p*¬*q* cases as counterevidence for it. Thus, people judge that not-*p* cases may be present or absent in the set. The uncertain categorical inference of not-*p* cases directly shows that categorical inferences are based on the pragmatic heuristic. This further implies the pragmatic heuristic of instance inferences that no available evidence can rule out that not-*p* cases are possible for the instance of the true conditional, and so they are judged possible for the instance. Thus, both the dominant instance and categorical inferences of not-*p* cases are due to the pragmatic heuristic that no available evidence can rule out not-*p* cases in the set and that not-*p* cases are possible for the instance. The pragmatic heuristic leads to the logical inconsistency between instance and categorical inferences of not-*p* cases. Overall, the pragmatic heuristic account can explain the inconsistent response pattern of not-*p* cases that is beyond the compatibility assumption in MMT. The compatibility assumption can explain instance inferences of not-*p* cases, but not categorical inferences of not-*p* cases. According to the compatibility assumption, judging not-*p* cases possible for the instance of the true conditional logically implies that not-*p* cases must be present in the set, violating the irrelevance assumption of not-*p* cases in suppositional theories. This implication was not the case. Thus, compared to the compatibility assumption, the heuristic account is a good explanation for the inconsistent response pattern of not-*p* cases.

In the inconsistent response pattern of not-*p* cases, instance inferences of not-*p* cases favor the prediction of ST1 over ST2, but categorical inferences of not-*p* cases favor only the prediction of ST2. We can adapt ST2 to the inconsistent response pattern of not-*p* cases by introducing the pragmatic heuristic to ST2. According to the extension of the pragmatic heuristic, what (*pq*, ¬*pq*, and ¬*p*¬*q*) no available evidence can rule out are possible, and what (*p*¬*q*) available evidence can rule out is impossible. Thus, for the present sampling task, the truth of an abstract suppositional conditional (*if p then q*) pragmatically implies that for instance inferences, each of the three cases (*pq*, ¬*pq*, and ¬*p*¬*q*) is possible for the instance of the conditional, but *p*¬*q* is impossible; and for categorical inferences, the set necessarily includes *pq* cases, and possibly includes irrelevant cases ¬*pq* and ¬*p*¬*q* cases, but impossibly includes *p*¬*q* cases. Here, irrelevant cases ¬*pq* and ¬*p*¬*q* may be present in the set, favoring the irrelevance assumption of not-*p* cases. This pragmatically implies that the true conditional doesn't require that ¬*pq* and ¬*p*¬*q* are possible for the instance of it. That the set must include *pq* cases but cannot include *p*¬*q* cases shows that these cases are relevant to the true conditional, and so implies that the true conditional requires that *pq* is possible for the instance, but *p*¬*q* is not. The overall inference pattern of relevant and irrelevant cases conforms to the constraint of the truth conditions of conditionals in ST2. Thus, this revised suppositional theory integrates the constraint of the truth conditions of conditionals in ST2 and the pragmatic heuristic. We call it ST3. ST3 accommodates the inconsistent response pattern of not-*p* cases, and so is not logically/semantically self-consistent. However, it is pragmatically self-consistent in that the pragmatic heuristic can explain the inconsistent response pattern of not-*p* cases.

In ST3, the pragmatic heuristic is generalizable to relevant cases *pq* and *p*¬*q*. Thus, the pragmatic heuristic account has the general implication that people judging each of *pq*, ¬*pq*, and ¬*p*¬*q* possible for the instance of a true conditional is essentially based the pragmatic heuristic, but not the semantic compatibility assumption in MMT. For the present sampling task, dominant instance modal inferences that can be explained by the semantic compatibility assumption can all be explained by the pragmatic heuristic, but uncertain categorical modal inferences of not-*p* cases that can be explained by the pragmatic heuristic cannot be explained by the semantic compatibility assumption. Thus, the pragmatic heuristic account with ST3 is a better explanation for instance and categorical modal inferences from conditionals, compared to the semantic compatibility assumption of MMT. The semantic compatibility assumption may be a superficial phenomenon based on the pragmatic heuristic. This suggests that the semantic compatibility assumption as the theoretical basis of MMT is essentially untenable. This suggestion is consistent with the empirical finding of the heuristic that for a true inclusive disjunction, *p or q*, people generally judge each of *pq*, ¬*pq*, and *p*¬*q* possible, due to that no available evidence can rule out that these cases are possible for the disjunction (Wang & Zheng, 2021).

In ST3, for the present sampling task with an abstract conditional, no available evidence can rule out *pq*, ¬*pq*, and ¬*p*¬*q*, and so they are pragmatically possible for the instance of the true conditional; and the true conditional as evidence can rule out *p*¬*q* and so *p*¬*q* is pragmatically impossible. Among the three possible cases, only the possibility of relevant *pq* is required by the true conditional and so only *pq* cases must be present in the set, but the two possibilities of irrelevant ¬*pq* and ¬*p*¬*q* is not required by the true conditional and so ¬*pq* and ¬*p*¬*q* cases may be absent or present in the set. Here, whether a possible case is required by the true conditional depends on whether it is relevant to the hypothetical test of the conditional. In ST3, according to the pragmatic heuristic account, instance modal inferences are pragmatic rather than semantic. The above pattern predicted by ST3 is confirmed by the present overall response pattern. The present overall response pattern refutes the predictions of MMT1 and MMT2. MMT1 predicts that each of the three possibilities is not required by the true conditional. MMT2 predicts that each of the three possibilities is required by the true conditional. MMT1 and MMT2 both hold that instance modal inferences are based on the semantic compatibility assumption and so are semantic. The semantic compatibility assumption predicts that ¬*pq* and ¬*p*¬*q* both are semantically possible for the instance of the true conditional and so must be present in the set. Thus, it is unable to explain why the two possibilities of ¬*pq* and ¬*p*¬*q* among the three possibilities is not required by the true conditional and so ¬*pq* and ¬*p*¬*q* cases may be absent in the set. Thus, compared to the semantic compatibility assumption, the pragmatic heuristic account is a better explanation for instance modal inferences from conditionals. Instance modal inferences from conditionals are more likely based on the pragmatic heuristic. Overall, ST3 with the pragmatic heuristic account is a better explanation for instance and categorical modal inferences from conditionals, compared to MMT1 and MMT2.

Moreover, the inconsistent response pattern of not-*p* cases refutes the assumption of MMT2 that a compound assertion means the conjunction of possibilities that it refers to. For the sampling problem, this assumption implies that the set must include all the three possible cases compatible with the true conditional. However, this was not the case that not-*p* cases may be present or absent in the set. The categorical inference favors the irrelevance assumption of not-*p* cases in suppositional theories over the assumption of MMT2.

Relevant cases *pq* and *p*¬*q* tended to elicit consistent response patterns. The consistent response pattern of *pq* cases conforms to Rule 1. The consistent response pattern of *p*¬*q* cases conforms to Rule 2. These two consistent response patterns support the basic logical principle that whether or not a set includes some case determines whether or not an instance that was randomly drawn from the set is possibly this case. Here, for relevant cases, people show the logical rationality of solving the sampling problems.

In the present experiment, instance inference questions yielded the same response pattern of the four cases as previous individual possibility judgement questions (Barrouillet, Gauffroy, & Lecas, 2008; Barrouillet, Grosset, & Lecas, 2000; Goodwin, 2014; Handley, Evans & Thompson, 2006; Schroyens, 2010; Wang, Yin, & Zheng, 2018). People generally judge each of the three cases (*pq*, ¬*pq*, and ¬*p*¬*q*) possible for the instance of a true conditional, but *p*¬*q* impossible. In the present instance inference questions with the three response options, most participants still judged not-*p* cases “possible” rather than “uncertain”. This tendency should be due to the pragmatic heuristic that no available evidence can rule out that not-*p* cases are possible. This pragmatic heuristic led to the inconsistent response pattern of not-*p* cases. Thus, instance inference questions are unable to determine whether not-*p* cases are required by the true conditional or not. In contrast, categorical inference questions are able to determine whether not-*p* cases are required or not, and distinguish between MMT2 and ST2. The present pattern of categorical inferences shows that *pq* cases are required by the true conditional and so they must be present in the set, whereas irrelevant cases ¬*pq* and ¬*p*¬*q* are not required by the true conditional and so they may be absent in the set. This categorical inference pattern favors the irrelevance assumption of not-*p* cases in suppositional theories, but not MMT2.

The dominant categorical inference that the set necessarily includes *pq* cases disfavors the new postulation of epistemic necessities that premises of a necessity state a necessary condition for another proposition (Ragni & Johnson-Laird, 2020, p. 5). According to the new postulation, in an inference from some premises to a conclusion, an epistemic necessity as the conclusion states a necessary condition for some other propositions to hold. When a statement as the conclusion is a necessary condition for some other proposition to hold, it is necessary; otherwise, it is not necessary. For example, consider the following argument problem in the paper:

> It's raining.
>
> If it's raining then it is necessary that it is warm.
>
> Does it follow that it is necessary that it is warm?

The two premises and the conclusion don’t state a necessary condition for some other unstated case, such as: It is necessary that it is warm *for the fruit to ripen*. Thus, reasoners should tend not to infer the conclusion that it is necessary that it is warm. For the present categorical inference task, the new postulation predicts that the set does not necessarily include *pq* cases because that the set includes *pq* cases is not a necessary condition for some other propositions to hold. Yet, this prediction was contrary to the case that participants tended to judge that the set necessarily includes *pq* cases. Moreover, participants tended to judge that the set necessarily excludes *p¬q* cases. This necessity conclusion is not a necessary condition for some other propositions to hold either, and so also disfavors the prediction of the new postulation.

In Experiments 4 and 5 in Ragni & Johnson-Laird (2020), they used only items without necessary conditions and they did not manipulate whether or not the conclusions were necessary conditions for some other propositions. Thus, the two experiments are unable to test whether or not a conclusion is a necessary condition for some other proposition determines whether or not the conclusion is necessary. Thus, the results do not directly support the new postulation of epistemic necessities. Moreover, the new postulation violates the logical validity principle that a conclusion is valid when it is true in any case in which the premises are true (Jeffrey, 1981). Logically, the validity of a conclusion depends on its premises, but not whether or not it is a necessary condition for some other propositions. Overall, the new postulation of epistemic necessities is empirically and logically untenable. Moreover, the new postulation is inconsistent with the original model theory of modal reasoning (Bell & Johnson-Laird, 1998), in which a conclusion is necessary if it holds in all the mental models of the premises. The introduction of the new postulation implies that MMT is not self-consistent on the whole.

Conclusion

In summary, current study investigates the relationship between instance and categorical modal inferences from conditionals. There are the following findings: (1) relevant cases *pq* and *p¬q* tend to elicit consistent response patterns conforming to Rules 1 and 2, but irrelevant cases ¬*pq* and ¬*p*¬*q* tend to elicit inconsistent response patterns violating Rules 1 and 3; (2) the inconsistent response pattern of not-*p* cases can be explained by the pragmatic heuristic that no evidence can rule out that not-*p* cases are possible; (3) the overall pattern of instance and categorical inferences is beyond each existing theory. Alternatively, ST3 integrating the constraint of the truth conditions of conditionals in ST2 and the pragmatic heuristic can explain the overall pattern of instance and categorical inferences. Overall, a true conditional requires that *pq* is possible and *p*¬*q* is impossible for the instance of the true conditional, but not that irrelevant not-*p* cases are possible. This favors the pragmatic heuristic account in ST3 over the semantic compatibility assumption in MMT.

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